EFFECT OF VOLUMED FOG FORMATION ON THE CONDENSATION OF SMALL VAPOR ADMIXTURES IN A GAS STREAM

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Analyzed is the effect of volumed fog formation on the precipitation of small vapor admixtures from a laminar gas stream at the cooled surface of a plate. It is shown that precipitation in this case may be different from diffusive precipitation.

Several factors govern the precipitation of small vapor admixtures on the cooled surface of a plate immersed lengthwise in a stream of noncondensating gas. These factors include vapor diffusion through the boundary layer [1], thermodiffusion [2], and volume fog formation [3]. This article will deal with the effect of volumed fog formation on vapor precipitation. The presence of a vapor supersaturation zone is evident from a comparison between the curves of saturation pressure and partial vapor pressure versus extra vapor pressure in the boundary layer. The solution to the problem in terms of the hydrodynamic, the thermal, and the diffusion boundary layers at the plate surface was given in [1] as the solution to the following system of ordinary differential equations:

$$\varphi''' + \varphi\varphi'' = 0; \tag{1'}$$

$$h'' + \mathrm{Pr}_\infty \varphi h' = 0; \tag{1''}$$

$$c'' + \mathrm{Pr}_{D_\infty} \varphi c' = 0 \tag{1'''}$$

with the boundary conditions $\varphi(0) = 0$, $\varphi(\infty) = 2$, $h(0) = h_W$, $h(\infty) = 1$, $c(0) = c_W$, $c(\infty) = 1$.

Equation (1‴) was derived from the corresponding partial differential equation for the case of a low vapor content in the gas. For this reason, then, the Stefan flow has been disregarded here. In this equation $\varphi(\zeta)$ is a function obtained from the Howarth function [4] by simple transformation and it is related to the flow function as follows:

$$\psi = \sqrt{\xi}\,\varphi(\zeta), \tag{2}$$

where

$$\zeta = \frac{\eta}{2\sqrt{\xi}}. \tag{3}$$

In order to account for the variability of thermophysical properties* in terms of coordinates ξ, η, it is convenient to use the A. A. Dorodnitsyn variables

$$\xi = x; \quad \eta = \int_0^y \rho' dy. \tag{4}$$

The solutions of Eqs. (1'), (1"), (1‴) are given in the form

$$\frac{u}{U_\infty} = \frac{1}{2}\varphi(\zeta), \tag{5}$$

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*The system of Eqs. (1'), (1"), (1‴) was solved on the assumption that the viscosity coefficients, the thermal conductivity, and the D_{12}/T ratio increase linearly with the temperature. Actually, these relations are somewhat weaker than that.

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$$\frac{T - T_W}{T_\infty - T_W} = \frac{\int\limits_0^\zeta [\varphi''(\zeta)]^{\mathrm{Pr}_\infty} d\zeta}{\int\limits_0^\infty [\varphi''(\zeta)]^{\mathrm{Pr}_\infty} d\zeta}, \tag{6}$$

$$\frac{P - P_W}{P_\infty - P_W} = \frac{\int\limits_0^\zeta [\varphi''(\zeta)]^{\mathrm{Pr}_{D\infty}} d\zeta}{\int\limits_0^\infty [\varphi''(\zeta)]^{\mathrm{Pr}_{D\infty}} d\zeta}. \tag{7}$$

For a specific example, with T_W, T_∞, P_W, P_∞, P_W, P_∞, Pr_∞, and $\mathrm{Pr}_{D\infty}$ known, it is possible, after the numerical values of the integrals in (6) and (7) have been found, to obtain the profiles of vapor temperature and concentration in the boundary layer. From the known relation $P_S = P_S(T)$, we can also plot the vapor saturation pressure as a function of the temperature in the boundary layer:

$$\bar{P}_s(T) = \frac{P_s(T) - P_W}{P_\infty - P_W}. \tag{8}$$

Here P_W denotes the vapor saturation pressure at a temperature equal to the wall temperature.

For example, the problem was solved for an air stream containing an admixture of water vapor and flowing around a plate at a velocity of 10 m/sec under a total pressure in the stream $\mathscr{P} = 1$ atm. The stream temperature was assumed equal to 50°C, the Prandtl number equal to 0.711, the Prandtl diffusion number (Schmidt number) equal to 0.616, the partial pressure of water vapor in air $P_\infty = 0.11$ kgf/cm^2, and the plate length L = 1 m. The solution to (7) and (8) for this case is shown in Fig. 1.

The solution was obtained for wall temperatures $T_W = -20$, -10, and 0°C.

The graph shows that near the wall saturation pressure drops below the partial pressure of vapor. This means that there exists a supersaturation zone and volumed fog formation occurs. Fog formation in the boundary layer causes a change in the concentration profile here. In order to determine the new concentration profile of the boundary layer, it is necessary to solve Eq. (1‴) so that, with some point $M(\zeta, P)$ as the origin, the boundary conditions $\bar{P}(\infty) = 1$ and $\bar{P}'(\infty) = 0$ are also satisfied. For such a solution one may use the same function $\varphi(\zeta)$ which was used for solving system (1), inasmuch as the hydrodynamic boundary layer is assumed not to change in the process.

We will represent Eq. (1‴) in terms of the following system of two linear differential equations:

$$c_1' = c_2, \tag{9'}$$

$$c_2' = -\mathrm{Pr}_{D_\infty} \varphi(\zeta) c_2. \tag{9''}$$

This system was solved numerically on a digital computer "PROMIN'". The location of point $M(\zeta_M, c_M)$ on curve $\bar{P}_S(\zeta)$ was selected so as to satisfy the condition

$$c_1(\infty) = 1; \quad c_2(\infty) = 0. \tag{10}$$

The slope at point M of the solution and at point M of the $\bar{P}_S(\zeta)$ curve were considered equal, moreover, so as to ensure continuity of the diffusion current. Near the wall, where the partial pressure varies linearly, the molar mass transfer is negligibly small. This decrease of the slope past point M indicates that part of the vapor stream becomes a fog.

Since even small differences between the calculated location of point M and its sought actual location caused large deviations from condition (10), hence it was possible to rather accurately establish the coordinates of point M. Determined were also $c_1(M)$ and $c_2(M)$.

This modified concentration boundary layer is shown in Fig. 1 by the dashed line.

We note that this method of solving the problem has yielded a larger value for the derivative of partial pressure with respect to coordinate at point M and a correspondingly larger diffusion current of vapor than in the case of no volumed fog formation. * This is indicated by the following values of the ratio

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*We assume that the number of condensation centers in the stream is sufficiently large and that fog formation starts before saturation has been reached. The larger diffusion current is due to an earlier vapor drain here than at the wall [5].

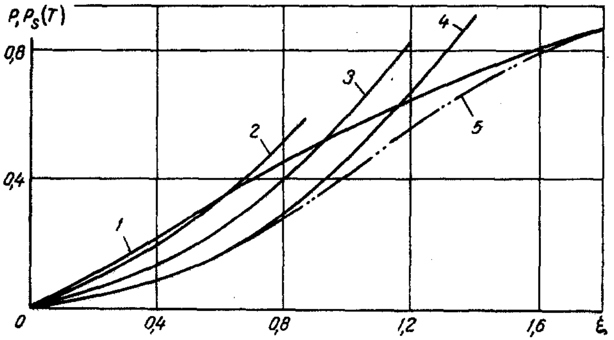

Fig. 1. Partial pressure $\bar{P}$ and equilibrium pressure $\bar{P}_s$ of vapor in the boundary layer, as a function of the dimensionless ζ coordinate: dimensionless partial vapor pressure in the boundary layer (1), dimensionless equilibrium vapor pressure in the boundary layer at a wall temperature $T_W = 0°C$ (2), $-10°C$ (3), $-20°$ (4), boundary layer modified by volumed fog formation (5).

$$T_w, °C = 0°; \ -10°; \ -20°; \quad \frac{c_2(M)}{c_2(0)} \doteq 1.08; \ 1.13; \ 1.23,$$

respectively.

The resulting aerosol is apparently a mist of submicron size particles which, under such conditions for small aerosol particles and, particularly, under a temperature gradient, obeys the equation of thermophoretic flow [6]. Under the conditions of our problem, therefore, an aerosol particle formed in the boundary layer moves with the stream at a velocity u at a given location in the boundary layer, and in the direction opposing the temperature gradient (in our case toward the cooled wall surface) at a thermophoretic velocity V_T. For $\delta < \lambda$, the latter is determined from the ratio

$$V_T \doteq -\frac{15k\mu \operatorname{grad} T}{20\mathcal{P}\left(1 + \frac{\pi\alpha}{8}\right) m_g}. \tag{11}$$

The equation of the trajectory is then

$$\frac{dy}{V_T} = \frac{dx}{u}. \tag{12}$$

This equation can be solved with the values of ζ_M, c_M, and c'_M found from the solution to (1'''), if a change is made to physical variables according to relations (3) and (4) and with known values of the physical parameters ρ, ρ_∞, ν, ν_∞, w_∞, etc. The relation between the physical variables will then be obtained in the form

$$y = 0.0077\sqrt{x}\,\zeta. \tag{13}$$

For specific values of coordinate ζ we will have a specific form of the relation between y and x. Taking into account also the nearly linear variation of velocity and temperature near the wall, we approximate the relations obtained as a result of a numerical solution according to formulas (5) and (6):

$$u = 0.86 \cdot 10^3 \frac{y}{\sqrt{x}}, \tag{14}$$

$$T = (-20 + 41.24\zeta) + 273.16, \tag{15'}$$

$$T = (-10 + 35.33\zeta) + 273.16, \tag{15''}$$

$$T = 29.45\zeta + 273.16 \tag{15'''}$$

respectively for wall temperatures $T_W = -20$, -10, and $0°C$. Such an approximation makes it possible to calculate the thermophoretic velocity and to solve the equations of aerosol trajectory.

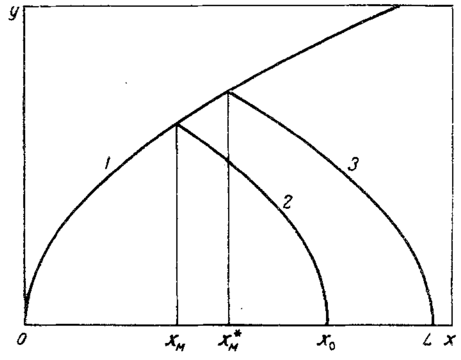

Fig. 2. Trajectories of aerosol particles: upper boundary of aerosol formation (1), trajectory of an aerosol droplet (2), extreme trajectory of an aerosol droplet which has reached the plate (3).

The solution to these equations are parabolas:

$$1.62\ 10^6 y^2 = x + A, \tag{16'}$$

$$1.46\ 10^6 y^2 = x + A, \tag{16''}$$

$$1.15\ 10^6 y^2 = x + A \tag{16'''}$$

for the respective wall temperatures $T_W = -20, -10$, and 0°C.

Constant A is determined from the stipulation that the initial coordinate of the particle is $y = y_M$ and its final coordinate is $y = 0$. One can also find a relation between x_M and x_0 of an aerosol particle (i. e., between the coordinate of the aerosol source and the coordinate of aerosol precipitation):

$$x_M = \frac{1}{B} x_0, \tag{17}$$

where x_M is obtained from expression (13) with a specific value given for ξ_M. Point x_M (Fig. 2) will correspond to this extreme trajectory of a particle which has reached the plate. Coefficient B we find by simultaneously solving the trajectory equations (16) and (13).

When no volumed condensation occurs, then the condensating substance precipitates on a given plate length in a quantity

$$G_{\text{diff}} = \int_0^L \left| -\frac{D_{12}M_1}{RT_\infty} \left(\frac{dP}{dy}\right)_{y=0} \right| dx. \tag{18}$$

When volumed condensation occurs, then the quantity of precipitate is

$$G' = \int_0^{x_M^*} k \left| -\frac{D_{12}M_1}{RT_\infty} \left(\frac{dP_s}{dy}\right)_{y=y_M} \right| dx + \int_{x_M^*}^{L} \left| -\frac{D_{12}M_1}{RT_\infty} \left[\left(\frac{dP_s}{dy}\right)_{y=\varphi(x)} - \left(\frac{dP}{dy}\right)_{y=0} \right] \right| dx + \int_0^L \left| -\frac{D_{12}M_1}{RT_\infty} \left(\frac{dP_s}{dy}\right)_{y=0} \right| dx, \tag{19}$$

where

$$k = \frac{\left(\frac{dP}{dy}\right)_{y=y_M} - \left(\frac{dP}{dy}\right)_{y=0}}{\left(\frac{dP}{dy}\right)_{y=y_M}}.$$

The first term in (19) represents the quantity of condensate which has penetrated into the boundary layer from the outer edge of the fog formation zone down to point x_M^* off the plate surface. The second term in this sum represents the quantity of aerosol formed behind point x_M^* within the fog formation zone. This quantity of aerosol precipitates on the surface. The third term of the sum represents the quantity of vapor precipitated on the surface. This portion of the vapor precipitates in accordance with the laws of diffusion. If dP_s/dy is assumed to vary linearly through the interval $[0, y_M]$, then the right-hand side of Eq. (19) can be rewritten as:

$$G' = \int_0^L \left| -\frac{D_{12}M_1}{RT_\infty}\left(\frac{dP_s}{dy}\right)_{y=y_M}\right| dx - \int_{x_M^*}^{L} \left| -\frac{D_{12}M_1}{RT_\infty}\left(\frac{dP_s}{dy}\right)_{y=0}\right| dx. \qquad (20)$$

The ratio G'/G_{diff} will be the criterion characterizing the effect which aerosol formation and aerosol ejection beyond the boundary layer has on the precipitation of small vapor admixtures from a gas stream.

Calculation based on the specific values here have yielded the following values for this ratio:

$$T_{\mathrm{w}},\ ^\circ\mathrm{C} = 0;\ -10;\ -20^\circ;\quad \frac{G'}{G_{\mathrm{diff}}} = 0.30;\ 0.17;\ 0.14.$$

At small temperature gradients and high stream velocities, in other words, the ejection of aerosol may be the predominant factor which abates the precipitation of vapor on a surface. The results may be different under other conditions. For example, $G'/G_{\mathrm{diff}} = 1$ during precipitation of K_2CO_3 and K_2SO_4 admixtures from a high-temperature stream of flue gases.

NOTATION

φ	is the function numerically equal to twice the dimensionless velocity in the boundary layer;
h	is the dimensionless temperature in the boundary layer;
c	is the dimensionless mass concentration in the boundary layer;
ψ	is the flow function;
ξ	is the dimensionless coordinate;
x, y	are the longitudinal and transverse coordinates respectively;
ρ	is the density of gas;
ρ'	is the gas density referred to density of oncoming stream;
T	is the temperature;
u, v	are the velocity components along the x and y axes respectively
T_∞, U_∞, P_∞	are the temperature, velocity, and partial pressure of vapor admixture in oncoming stream;
T_W, P_W	are the temperature and partial pressure of vapor at the wall surface;
ρ_∞	is the density of oncoming stream;
$D_{12\infty}$	is the diffusivity of oncoming stream;
μ_∞	is the dynamic viscosity of oncoming stream;
λ_∞	is the thermal conductivity of oncoming stream;
a_∞	is the thermal diffusivity of oncoming stream;
$Pr_{D\infty} = \nu_\infty/D_{n\infty}$	is the Prandtl diffusion number (Schmidt number) referred to oncoming stream;
$Pr_\infty = \nu_\infty/a_\infty$	Prandtl number referred to oncoming stream;
$k = 1.38 \cdot 10^{-23}\ m^2 \cdot kg/sec^2 \cdot {}^\circ K$	is the Boltzmann constant;
$\alpha \approx 1$	is the fraction of diffusively dispersed particles;
$mg \approx 4.8 \cdot 10^{-26}$ kg	is the mass of gas molecule;
V_T	is the thermophoretic velocity;
λ	is the free-path-length of molecule in the gas
δ	is the characteristic dimension of aerosol particle;
$\bar{P} = (P-P_W)/(P-P_W)$	is the dimensionless partial pressure of vapor.

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